Bootstrap for Gravity Models
(preliminary)

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Conference in Honor of Jeff Bergstrand

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This Paper

- Continues the agenda from Weidner and Zylkin JIE 2021
  - Estimates for gravity models with two- (or three-)way fixed effects gravity models are biased
  - How can we get more reliable inferences?
- Gravity: workhorse model in trade for estimating effects of trade policies (thanks Jeff!)
- Idea: we can use the **bootstrap** to remove bias
  - How? Why? Which bootstrap method(s) should we use?
  - How does it work? (theory)

Motivation

Table: Recapping some results from Weidner and Zylkin (2021)

	N=20			N=50			N=100		
	T=2	T=5	T=10	T=2	T=5	T=10	T=2	T=5	T=10
II. Poisson DGP									
Coverage probability with uncorrected SEs (should be 0.95 for an unbiased estimator)									
FE-PPML	0.887	0.880	0.892	0.912	0.905	0.919	0.918	0.919	0.925
Analytical BC	0.888	0.897	0.902	0.920	0.931	0.938	0.934	0.939	0.948
Jackknife BC	0.857	0.870	0.884	0.916	0.922	0.934	0.928	0.936	0.945
Coverage probability with corrected SEs (should be 0.95 for an unbiased estimator)									
(uncorrected)	0.887	0.880	0.892	0.912	0.905	0.919	0.918	0.919	0.925
FE-PPML + HC2 SEs	0.923	0.915	0.916	0.927	0.921	0.930	0.925	0.927	0.931
Analytical BC + HC2 SEs	0.923	0.929	0.930	0.938	0.942	0.949	0.942	0.945	0.952
Jackknife BC + HC2 SEs	0.900	0.903	0.915	0.932	0.935	0.942	0.936	0.941	0.949

Model: $y_{ijt} = \exp(\alpha_{it} + \gamma_{jt} + \eta_{ij} + \beta x_{ijt})\omega_{ijt}$ ("three-way gravity") N: no. countries. T: time periods. Estimator: PPML.

Weidner and Zylkin (2021) show that "three-way" PPML gravity estimates are consistent, **BUT**:

1. Estimates are asymptotically biased due to the incidental parameter problem
2. Standard errors are downward biased as well.
3. Using corrections for both the estimates and SEs can improve inferences

Motivation

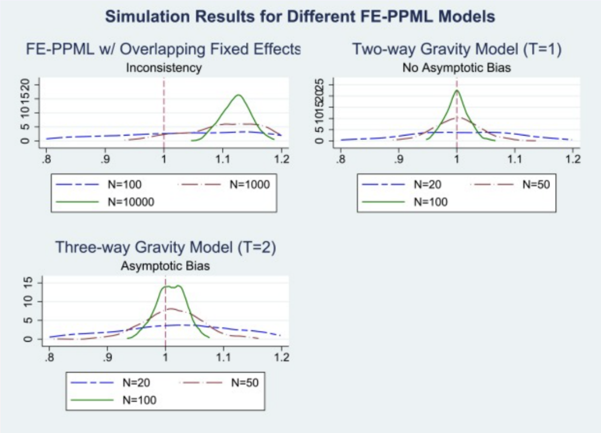
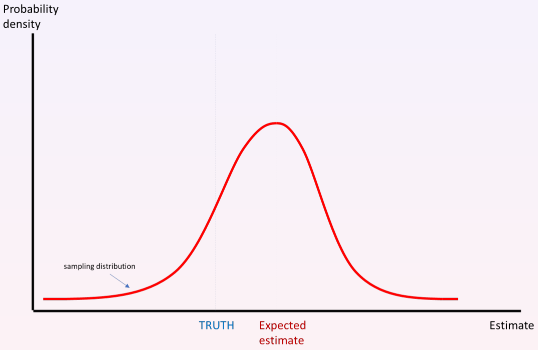


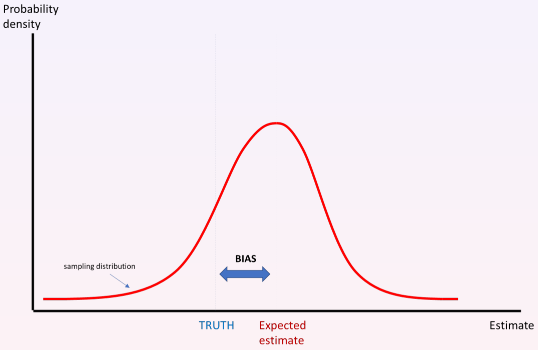
Figure: Figure from Weidner and Zylkin (2021) illustrating "asymptotic bias"



Idea behind bootstap bias correction

Left: sampling distribution of a **biased** estimator

Idea: the bootstrap as a bias correction method



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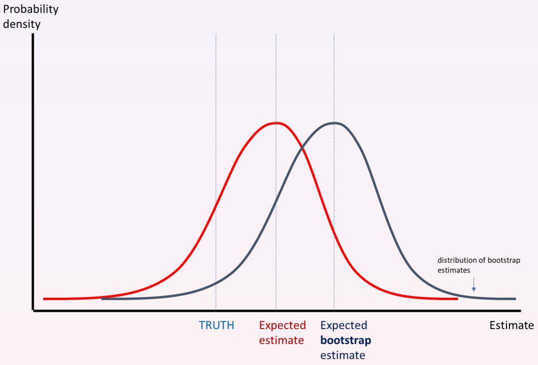


Idea behind bootstap bias correction

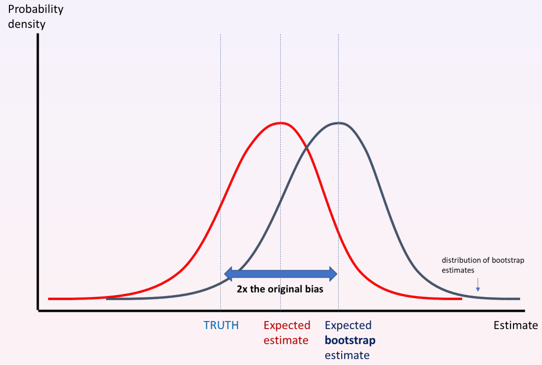
Left: sampling distribution of a biased estimator

When we **bootstrap** the data, the bootstrap samples are drawn from a "population" where the biased estimate is the "truth".

So:

1. The bias of each bootstrap estimate is 2x that of the original estimate
2. We can estimate the bias by comparing the average bootstrap estimate with the original estimate.

Idea: the bootstrap as a bias correction method



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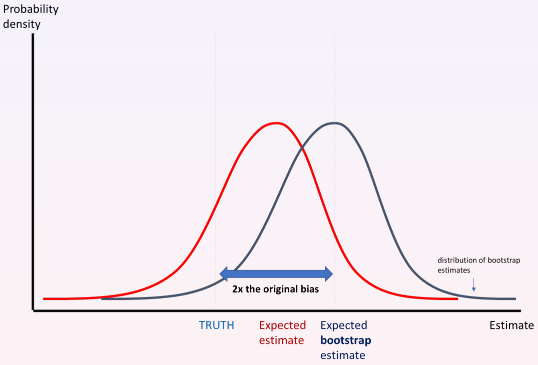

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1. The bias of each bootstrap estimate is 2x that of the original estimate
2. We can **estimate the bias** by comparing the average bootstrap estimate with the original estimate.

Why bootstrap?

Even though there often exist other alternatives, bootstrap bias correction can be a good option!

- Potential for refinements along **two** margins using a single procedure
  - Bootstrap SEs seem to remove bias in confidence interval width (Pfaffermayr 2021)
- Very easy to implement analytically - only need the assumed sampling process
  - don't need to derive/code complicated formulas for the bias
  - don't even need to know the **order** of the bias! (needed for jackknife)
- computational efficiency can gained using k-step bootstrap (Kim and Sun 2016)

HOW?

HOW you bootstrap turns out to matter

The literature offers a lot of alternatives, e.g.

- Traditional re-sampling bootstrap ("pairs bootstrap")
- Parametric bootstrap
- Kline and Santos "wild score" bootstrap

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"Fractional weight" bootstrap ("Bayesian bootstrap")

Another option that has become popular recently:

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- Traditional re-sampling bootstrap ("pairs bootstrap") Generally works well!
- Parametric bootstrap Not suitable for PML
- Kline and Santos "wild score" bootstrap Not meant for removing bias!

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"Fractional weight" bootstrap ("Bayesian bootstrap")

Another option that has become popular recently:

Does surprisingly poorly compared to re-sampling approach!

Related literature

Bias of two-way and three-way fixed effects estimators

Fernandez-Val and Weidner (2016, 2018), Weidner and Zylkin (2021)

Bootstrap

- (foundations) Quenouille (1949), Efron (1979), Rubin (1981), Parr (1983), Hall (1992), Horowitz (2001)
- (bias correction for panel data models) Kim and Sun (2016), Hahn, Hughes, Kuersteiner, and Newey (2023), Higgins and Jochmans (2023)

Bias of "heteroskedasticity-robust" standard errors

- (in general) McKinnon and White (1985), Imbens and Kolesar (2016), Cameron, Gelbach, and Miller (2015), McKinnon, Nielson, and Webb (2023)
- (for PML gravity estimators) Egger and Staub (2015), Jochmans (2017), Pfaffermayr (2019), Weidner and Zylkin (2021)
- (conservativism of bootstrap SEs) Hahn and Liao (2021) (bootstrap SEs for gravity estimates) Pfaffermayr (2021)

Theory: Two-way gravity

Model

Suppose we have the following gravity model:

$$y_{ij} = \exp\left(\alpha_i + \gamma_j + x_{ij}\beta^0\right)\omega_{ij}$$

- β^0: parameter of interest (effect of distance, trade agreement,...)
- α_i, γ_j: exporter and importer fixed effects

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IPP bias in two-way FE models

From Fernandez-val and Weidner (2016), we know for two-way FE models that

$$E(\widehat{\beta}) = \beta^0 + \frac{1}{N}B_\alpha^\infty + \frac{1}{N}B_\gamma^\infty + \text{higher-order terms}$$

- B_α^∞, B_γ^∞: asymptotic bias terms due to estimation noise in $\widehat{\alpha}_i$, $\widehat{\gamma}_j$
- For any two-way FE estimator, $\widehat{\beta} \rightarrow_d \beta^0$ as $N \rightarrow \infty$ **(consistency)**

Theory: *Three-way* gravity

Model

For the *three-way* gravity model, we have

$$y_{ijt} = \exp\left(\alpha_{it} + \gamma_{jt} + \eta_{ij} + x_{ijt}\beta^0\right)\omega_{ijt}$$

- β^0: coefficient for *time-varying* trade cost variables (FTA)
- α_{it}, γ_{jt}, η_{ij}: exporter-time, importer-time and exporter-importer fixed effects

IPP bias of three-way FE PPML estimator

For three-way PPML, Weidner and Zylkin (2021) show the bias remains

$$E(\widehat{\beta}) = \beta^0 + \frac{1}{N}B_\alpha^\infty + \frac{1}{N}B_\gamma^\infty + \text{higher-order terms}$$

- B_α^∞, B_γ^∞: asymptotic bias terms due to estimation noise in $\widehat{\alpha}_{it}$, $\widehat{\gamma}_{jt}$ only
- Special property of PPML: can eliminate $\widehat{\eta}_{ij}$'s contribution to the bias (ensures consistency w/ fixed T)

Why bootstrap bias correction works (theory)

For the **original estimation**, we have:

Model:

$$E(y_{ij}|x_{ij},..) = \mu_{ij} := \exp(\alpha_i + \gamma_j + x_{ij}\beta)$$

PML estimation:

$$(\beta, \alpha, \gamma) = \arg\max_{\beta,\alpha,\gamma} \mathcal{L} := \sum_{i,j} \ell_{ij}\left(\beta, \alpha_i, \gamma_j\right)$$

- for **PPML**, $\ell_{ij} = y_{ij} \log \mu_{ij} - \mu_{ij}$
- for **Gamma PML**, $\ell_{ij} = y_{ij}/\mu_{ij} - \log \mu_{ij}$

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Bias (Fernandez-Val and Weidner 2016):

$$\mathbb{E}(\widehat{\beta} - \beta^0) \approx \frac{H^{-1}}{N-1}\Bigg(-\frac{1}{N}\sum_{i=1}^{N}\frac{\sum_j \mathbb{E}\left(\ell_{ij}^{*\beta_k\alpha_i}\,\ell_{ij}^{\alpha_i}\right)}{\sum_{j\neq i}^{N}\bar{\ell}_{ij}^{\alpha_i\alpha_i}} + \frac{1}{2\,N}\sum_{i=1}^{N}\frac{\left(\sum_{j\neq i}\bar{\ell}_{ij}^{*\beta_k\alpha_i\alpha_i}\right)\left[\sum_{j\neq i}\mathbb{E}\left(\ell_{ij}^{\alpha_i}\,\ell_{ij}^{\alpha_i}\right)\right]}{\left(\sum_j^N \bar{\ell}_{ij}^{\alpha_i\alpha_i}\right)^2}$$

$$-\frac{1}{N}\sum_{j=1}^{N}\frac{\sum_i \mathbb{E}\left(\ell_{ij}^{*\beta_k\gamma_j}\,\ell_{ij}^{\gamma_j}\right)}{\sum_{i\neq j}\bar{\ell}_{ij}^{\gamma_j\gamma_j}} + \frac{1}{2\,N}\sum_{j=1}^{N}\frac{\left(\sum_{i\neq j}\bar{\ell}_{ij}^{*\beta_k\gamma_j\gamma_j}\right)\left[\sum_{i\neq j}\mathbb{E}\left(\ell_{ij}^{\gamma_j}\,\ell_{ij}^{\gamma_j}\right)\right]}{\left(\sum_{i\neq j}\bar{\ell}_{ij}^{\gamma_j\gamma_j}\right)^2}\Bigg)$$

- an prder-1/N bias that depends on the partial derivatives and higher-order derivatives of ℓ_{ij}.
- same order as the standard error (biased inferences!)

Why bootstrap bias correction works (theory)

For each **bootstrap estimate** $b = 1, ..., B$, we have:

Model:

$$E(y_{ij}|x_{ij}, ..) = \mu_{ij} := \exp(\alpha_i + \gamma_j + x_{ij}\beta)$$

(weighted) PML estimation:

$$(\beta, \alpha, \gamma) = \arg\max_{\beta,\alpha,\gamma} \mathcal{L} := \sum_{i,j} W_{ij,b}\ell_{ij}\left(\beta, \alpha_i, \gamma_j\right)$$

- For the resampling bootstrap, each bootstrap weight $W_{ij,b}$ is a random integer (0, 1, 2, ...)
- For the fractional weight bootstrap, each $W_{ij,b}$ is a continuous random variable.
- In either case, $\mathbb{E}(W_{ij,b}) = Var(W_{ij,b}) = 1$.

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Question: What is $\mathbb{E}[W_{ij,b}^2]$?

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The asymptotic bias of each bootstrap estimate is:

$$\mathbb{E}(\widehat{\beta} - \beta^0) \approx \frac{H^{-1}}{N-1}\left(-\frac{1}{N}\sum_{i=1}^{N}\frac{\sum_j \mathbb{E}\left(W_{ij,b}^2 \ell_{ij}^{*\beta_k\alpha_i}\ \ell_{ij}^{\alpha_i}\right)}{\sum_{j\neq i}^{N} W_{ij,b}\bar{\ell}_{ij}^{\alpha_i\alpha_i}} + \frac{1}{2N}\sum_{i=1}^{N}\frac{\left(\sum_{j\neq i} W_{ij,b}\bar{\ell}_{ij}^{*\beta_k\alpha_i\alpha_i}\right)\left[\sum_{j\neq i}\mathbb{E}\left(W_{ij,b}^2\ell_{ij}^{\alpha_i}\ \ell_{ij}^{\alpha_i}\right)\right]}{\left(\sum_j^N W_{ij}^{(b)}\bar{\ell}_{ij}^{\alpha_i\alpha_i}\right)^2}\right.$$
$$\left.-\frac{1}{N}\sum_{j=1}^{N}\frac{\sum_i \mathbb{E}\left(W_{ij,b}^2 \ell_{ij}^{*\beta_k\gamma_j}\ \ell_{ij}^{\gamma_j}\right)}{\sum_{i\neq j} W_{ij,b}\bar{\ell}_{ij}^{\gamma_j\gamma_j}} + \frac{1}{2N}\sum_{j=1}^{N}\frac{\left(\sum_{i\neq j}\bar{\ell}_{ij}^{*\beta_k\gamma_j\gamma_j}\right)\left[\sum_{i\neq j}\mathbb{E}\left(W_{ij,b}^2\ell_{ij}^{\gamma_j}\ \ell_{ij}^{\gamma_j}\right)\right]}{\left(\sum_{i\neq j} W_{ij,b}\bar{\ell}_{ij}^{\gamma_j\gamma_j}\right)^2}\right)$$

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As $N \longrightarrow \infty$, we have

$$\mathbb{E}(\widehat{\beta} - \beta^0) \approx \frac{H^{-1}}{N-1}\left(-\frac{1}{N}\sum_{i=1}^{N}\frac{\sum_j \mathbb{E}\left(\underline{2}\,\ell_{ij}^{*\beta_k\alpha_i}\ \ell_{ij}^{\alpha_i}\right)}{\sum_{j\neq i}^{N}\overline{\ell}_{ij}^{\alpha_i\alpha_i}} + \frac{1}{2\,N}\sum_{i=1}^{N}\frac{\left(\sum_{j\neq i}\overline{\ell}_{ij}^{*\beta_k\alpha_i\alpha_i}\right)\left[\sum_{j\neq i}\mathbb{E}\left(\underline{2}\,\ell_{ij}^{\alpha_i}\ \ell_{ij}^{\alpha_i}\right)\right]}{\left(\sum_j^N \overline{\ell}_{ij}^{\alpha_i\alpha_i}\right)^2}\right.$$

$$\left. -\frac{1}{N}\sum_{j=1}^{N}\frac{\sum_i \mathbb{E}\left(\underline{2}\,\ell_{ij}^{*\beta_k\gamma_j}\ \ell_{ij}^{\gamma_j}\right)}{\sum_{i\neq j}\overline{\ell}_{ij}^{\gamma_j\gamma_j}} + \frac{1}{2\,N}\sum_{j=1}^{N}\frac{\left(\sum_{i\neq j}\overline{\ell}_{ij}^{*\beta_k\gamma_j\gamma_j}\right)\left[\sum_{i\neq j}\mathbb{E}\left(\underline{2}\,\ell_{ij}^{\gamma_j}\ \ell_{ij}^{\gamma_j}\right)\right]}{\left(\sum_{i\neq j}\overline{\ell}_{ij}^{\gamma_j\gamma_j}\right)^2}\right)$$

Each bootstrap estimate has two times the bias of the original estimate.

Other corrections

Analytical methods

Derive analytical formulas for the bias using Taylor expansions:

- Point estimates: Fernandez-val and Weidner (2016), Weidner and Zylkin (2021)
- "HC2" / "CR2" Standard errors: Weidner and Zylkin (2021)

Jackknife

For standard error corrections:

- each jackknife sample holds out one observation at a time
- compute "jackknife SEs" based on the standard deviation of the jackknife samples

For correcting point estimates:

- "N-jackknife": hold out one *country* at a time to inflate the $1/N$ bias
- "split-panel jackknife" (SPJ): hold out half the exporters/importers at time (4 subsamples)

Horse race!

For the *two-way* gravity model

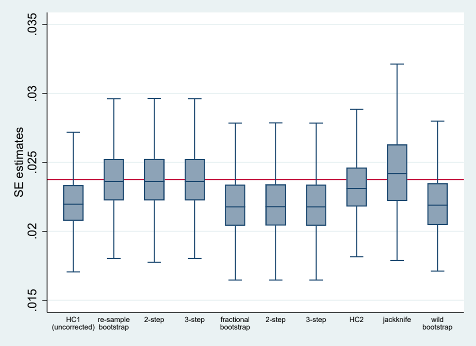
- Simulate $y_{ij} = \exp(\alpha_i + \gamma_j + x_{ij}\beta)\omega_{ij}$
- Estimate using both PPML and Gamma PML
- Standard error corrections (for *both* estimators): analytical ("HC2"), different flavors of bootstrap, jackknife
- Bias corrections (for Gamma *only*): analytical, different flavors of bootstrap, split-panel jackknife

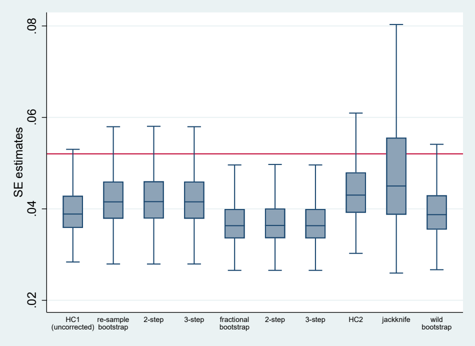
For the *three-way* gravity model

- Simulate $y_{ijt} = \exp(\alpha_{it} + \gamma_{jt} + \eta_{ij} + x_{ijt}\beta)\omega_{ijt}$
- Estimate using PPML only
- Experiment with different corrections for both the point estimates and the standard errors

For all simulations: 1000 replications, 1000 bootstrap draws per replication, $N = 50$ or 100

Simulation results: Standard errors for 2-way **PPML**



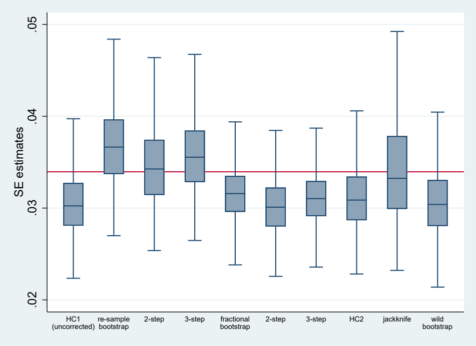


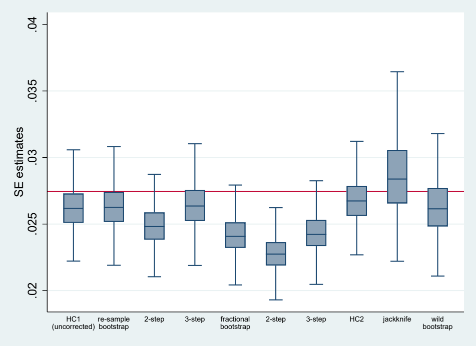
Left: PPML is correctly specified: $Var(\omega_{ij}) = \kappa\mu_{ij}$.

Right: Gamma PML is correctly specified: $Var(\omega_{ij}) = \kappa\mu_{ij}^2$.

The **red line** is the standard deviation of estimates across simulations

Simulation results: Standard errors for 2-way **Gamma PML**





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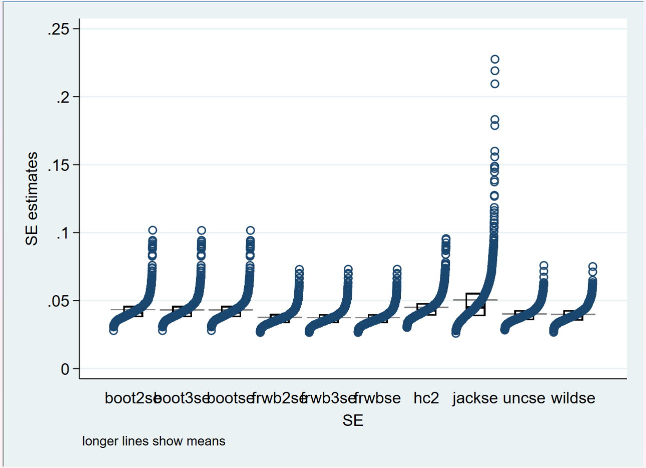
Jackknife SEs have very wide dispersion (*another look*)



This is a “strip plot” for PPML estimates from case 2.

Table: Improving coverage for two-way FE-PML gravity estimators (case 1)

	N=50				N=100			
	Bias	Bias/SD	SE/SD	95% Cov.	Bias	Bias/SD	SE/SD	95% Cov.
A. **PPML** (case 1)								
PPML, uncorrected	-0.001	-0.058	0.935	0.936	...	...	...	...
PPML with corrected SEs/CIs								
Bootstrap SEs	-0.001	-0.058	1.003	0.954	...	...	...	...
2-step bootstrap SEs	-0.001	-0.058	1.003	0.954				
FRW bootstrap SEs	-0.001	-0.058	0.926	0.938				
Jackknife SEs	-0.001	-0.058	1.032	0.955				
Analytical (HC2) SEs	-0.001	-0.058	0.982	0.950				
B. **Gamma PML** (case 1)								
Gamma PML, uncorrected	0.037	1.092	0.911	0.722	...	...	...	...
Re-centered Gamma PML								
Analytical BC	0.010	0.256	0.789	0.875	...	...	...	...
Bootstrap BC	0.016	0.438	0.850	0.870				
2-step Bootstrap BC	0.016	0.424	0.832	0.865				
FRW boot BC	0.020	0.569	0.867	0.854				
Split-panel Jackknife BC	0.011	0.293	0.828	0.884				
Node Jackknife BC	0.008	0.208	0.798	0.881				
Fully corrected Gamma PML (top 3 + selected others)								
SPJ + bootstrap SEs	0.011	0.293	0.993	0.932	...	...	...	...
Node J. + bootstrap SEs	0.008	0.208	0.957	0.926				
Analytical + bootstrap SEs	0.010	0.256	0.946	0.924				
Bootstrap + bootstrap SEs	0.016	0.438	1.018	0.918				
FRWB + FRWB SEs	0.020	0.569	0.891	0.865				
Analytical + HC2 SEs	0.010	0.256	0.806	0.884				
SPJ + Jackknife SEs	0.011	0.293	0.933	0.918				

Notes: 1,000 repetitions + 1,000 bootstrap trials per repetition. Model: $y_{ijt} = \exp(\alpha_i + \gamma_j + 0.5x_{ij})\omega_{ij}$. **Case 1 is the case where PPML is correctly specified.**

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Analytical (HC2) SEs	-0.001	-0.058	0.982	0.950				
B. **Gamma PML** (case 1)								
Gamma PML, uncorrected	0.037	1.092	0.911	0.722	...	...	...	...
Re-centered Gamma PML								
Analytical BC	0.010	0.256	0.789	0.875	...	...	...	...
Bootstrap BC	0.016	0.438	0.850	0.870				
2-step Bootstrap BC	0.016	0.424	0.832	0.865				
FRW boot BC	0.020	0.569	0.867	0.854				
Split-panel Jackknife BC	0.011	0.293	0.828	0.884				
Node Jackknife BC	0.008	0.208	0.798	0.881				
Fully corrected Gamma PML (top 3 + selected others)								
SPJ + bootstrap SEs	0.011	0.293	0.993	0.932	...	...	...	...
Node J. + bootstrap SEs	0.008	0.208	0.957	0.926				
Analytical + bootstrap SEs	0.010	0.256	0.946	0.924				
Bootstrap + bootstrap SEs	0.016	0.438	1.018	0.918				
FRWB + FRWB SEs	0.020	0.569	0.891	0.865				
Analytical + HC2 SEs	0.010	0.256	0.806	0.884				
SPJ + Jackknife SEs	0.011	0.293	0.933	0.918				

Notes: 1,000 repetitions + 1,000 bootstrap trials per repetition. Model: $y_{ijt} = \exp(\alpha_i + \gamma_j + 0.5x_{ij})\omega_{ij}$. **Case 1 is the case where PPML is correctly specified.**

Table: Improving coverage for two-way FE-PML gravity estimators (case 1)

	N=50				N=100			
	Bias	Bias/SD	SE/SD	95% Cov.	Bias	Bias/SD	SE/SD	95% Cov.
A. **PPML** (case 1)								
PPML, uncorrected	-0.001	-0.058	0.935	0.936	...	...	...	...
PPML with corrected SEs/CIs								
Bootstrap SEs	-0.001	-0.058	1.003	0.954	...	...	...	...
2-step bootstrap SEs	-0.001	-0.058	1.003	0.954				
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Table: Improving coverage for two-way FE-PML gravity estimators (case 1)

	N=50				N=100			
	Bias	Bias/SD	SE/SD	95% Cov.	Bias	Bias/SD	SE/SD	95% Cov.
A. **PPML** (case 1)								
PPML, uncorrected	-0.001	-0.058	0.935	0.936	.0006	.0474	.9377	.939
PPML with corrected SEs/CIs								
Bootstrap SEs	-0.001	-0.058	1.003	0.954	.0006	.0474	.9698	.944
2-step bootstrap SEs	-0.001	-0.058	1.003	0.954	.0006	.0474	.9699	.944
FRW bootstrap SEs	-0.001	-0.058	0.926	0.938	.0006	.0474	.9309	.934
Jackknife SEs	-0.001	-0.058	1.032	0.955	.0006	.0474	.9878	.945
Analytical (HC2) SEs	-0.001	-0.058	0.982	0.950	.0006	.0474	.9633	.947
B. **Gamma PML** (case 1)								
Gamma PML, uncorrected	0.037	1.092	0.911	0.722	.0225	1.2356	.9274	.695
Re-centered Gamma PML								
Analytical BC	0.010	0.256	0.789	0.875	.0051	.2479	.8232	.874
Bootstrap BC	0.016	0.438	0.850	0.870	.0087	.4464	.871	.870
2-step Bootstrap BC	0.016	0.424	0.832	0.865	.0080	.4030	.8533	.873
FRW boot BC	0.020	0.569	0.867	0.854	.0051	.2479	.9154	.917
Split-panel Jackknife BC	0.011	0.293	0.828	0.884	.0063	.3224	.8609	.880
Node Jackknife BC	0.008	0.208	0.798	0.881	.0044	.2167	.8292	.882
Fully corrected Gamma PML (top 3 + selected others)								
SPJ + bootstrap SEs	0.011	0.293	0.993	0.932	...	...	...	...
Node J. + bootstrap SEs	0.008	0.208	0.957	0.926				
Analytical + bootstrap SEs	0.010	0.256	0.946	0.924				
Bootstrap + bootstrap SEs	0.016	0.438	1.018	0.918				
FRWB + FRWB SEs	0.020	0.569	0.891	0.865				
Analytical + HC2 SEs	0.010	0.256	0.806	0.884				
SPJ + Jackknife SEs	0.011	0.293	0.933	0.918				

Notes: 1,000 repetitions + 1,000 bootstrap trials per repetition. Model: $y_{ijt} = \exp(\alpha_i + \gamma_j + 0.5x_{ij})\omega_{ij}$. **Case 1 is the case where PPML is correctly specified.**

Table: Improving coverage for two-way FE-PML gravity estimators (case 1)

	N=50				**N=100**			
	Bias	Bias/SD	SE/SD	95% Cov.	Bias	Bias/SD	SE/SD	95% Cov.
A. **PPML** (case 1)								
PPML, uncorrected	-0.001	-0.058	0.935	0.936	...	...	...	...
PPML with corrected SEs/CIs								
Bootstrap SEs	-0.001	-0.058	1.003	0.954	...	...	...	...
2-step bootstrap SEs	-0.001	-0.058	1.003	0.954				
FRW bootstrap SEs	-0.001	-0.058	0.926	0.938				
Jackknife SEs	-0.001	-0.058	1.032	0.955				
Analytical (HC2) SEs	-0.001	-0.058	0.982	0.950				
B. **Gamma PML** (case 1)								
Gamma PML, uncorrected	0.037	1.092	0.911	0.722	...	...	...	...
Re-centered Gamma PML								
Analytical BC	0.010	0.256	0.789	0.875	...	...	...	...
Bootstrap BC	0.016	0.438	0.850	0.870				
2-step Bootstrap BC	0.016	0.424	0.832	0.865				
FRW boot BC	0.020	0.569	0.867	0.854				
Split-panel Jackknife BC	0.011	0.293	0.828	0.884				
Node Jackknife BC	0.008	0.208	0.798	0.881				
Fully corrected Gamma PML (top 3 + selected others)								
SPJ + bootstrap SEs	0.011	0.293	0.993	0.932	...	...	...	...
Node J. + bootstrap SEs	0.008	0.208	0.957	0.926				
Analytical + bootstrap SEs	0.010	0.256	0.946	0.924				
Bootstrap + bootstrap SEs	0.016	0.438	1.018	0.918				
FRWB + FRWB SEs	0.020	0.569	0.891	0.865				
Analytical + HC2 SEs	0.010	0.256	0.806	0.884				
SPJ + Jackknife SEs	0.011	0.293	0.933	0.918				

Notes: 1,000 repetitions + 1,000 bootstrap trials per repetition. Model: $y_{ijt} = \exp(\alpha_i + \gamma_j + 0.5x_{ij})\omega_{ij}$. **Case 1 is the case where PPML is correctly specified.**

Table: Improving coverage for two-way FE-PML gravity estimators (case 1)

	N=50				N=100			
	Bias	Bias/SD	SE/SD	95% Cov.	Bias	Bias/SD	SE/SD	95% Cov.
A. **PPML** (case 1)								
PPML, uncorrected	-0.001	-0.058	0.935	0.936	0.001	0.047	0.938	0.939
PPML with corrected SEs/CIs								
Bootstrap SEs	-0.001	-0.058	1.003	0.954	0.001	0.047	0.9698	0.944
2-step bootstrap SEs	-0.001	-0.058	1.003	0.954	0.001	0.047	0.9699	0.944
FRW bootstrap SEs	-0.001	-0.058	0.926	0.938	0.001	0.047	0.9309	0.934
Jackknife SEs	-0.001	-0.058	1.032	0.955	0.001	0.047	0.9878	0.945
Analytical (HC2) SEs	-0.001	-0.058	0.982	0.950	0.001	0.047	0.9633	0.947
B. **Gamma PML** (case 1)								
Gamma PML, uncorrected	0.037	1.092	0.911	0.722	0.023	1.236	0.927	0.695
Re-centered Gamma PML								
Analytical BC	0.010	0.256	0.789	0.875	0.005	0.248	0.823	0.874
Bootstrap BC	0.016	0.438	0.850	0.870	0.009	0.446	0.871	0.870
2-step Bootstrap BC	0.016	0.424	0.832	0.865	0.008	0.403	0.853	0.873
FRW boot BC	0.020	0.569	0.867	0.854	0.011	0.564	0.915	0.917
Split-panel Jackknife BC	0.011	0.293	0.828	0.884	0.006	0.322	0.861	0.880
Node Jackknife BC	0.008	0.208	0.798	0.881	0.004	0.217	0.829	0.882
Fully corrected Gamma PML (top 3 + selected others)								
SPJ + bootstrap SEs	0.011	0.293	0.993	0.932	0.006	0.322	1.078	0.946
Node J. + bootstrap SEs	0.008	0.208	0.957	0.926	0.004	0.217	1.038	0.952
Analytical + bootstrap SEs	0.010	0.256	0.946	0.924	0.005	0.248	1.031	0.945
Bootstrap + bootstrap SEs	0.016	0.438	1.018	0.918	0.009	0.446	1.091	0.938
FRWB + FRWB SEs	0.020	0.569	0.891	0.865	0.011	0.564	0.985	0.892
Analytical + HC2 SEs	0.010	0.256	0.806	0.884	0.005	0.248	0.832	0.876
SPJ + Jackknife SEs	0.011	0.293	0.933	0.918	0.006	0.322	0.893	0.874

Notes: 1,000 repetitions + 1,000 bootstrap trials per repetition. Model: $y_{ijt} = \exp(\alpha_i + \gamma_j + 0.5x_{ij})\omega_{ij}$. **Case 1 is the case where PPML is correctly specified.**

Table: Improving coverage for two-way FE-PML gravity estimators (**case 2**)

	N=50				N=100			
	Bias	Bias/SD	SE/SD	95% Cov.	Bias	Bias/SD	SE/SD	95% Cov.
A. **PPML** (case 2)								
PPML, uncorrected	-0.003	-0.054	0.770	0.874	0.003	0.011	0.845	0.906
PPML with corrected SEs/CIs								
Bootstrap SEs	-0.003	-0.054	0.832	0.906	0.003	0.011	.8589	.917
2-step bootstrap SEs	-0.003	-0.054	0.833	0.906	0.003	0.011	.8595	.917
FRW bootstrap SEs	-0.003	-0.054	0.720	0.848	0.003	0.011	.783	.882
Jackknife SEs	-0.003	-0.054	0.969	0.911	0.003	0.011	.9167	.886
Analytical (HC2) SEs	-0.003	-0.054	0.866	0.911	0.003	0.011	.9049	.927
B. **Gamma PML** (case 2)								
Gamma PML, uncorrected	-0.001	-0.023	0.958	0.943	0.0004	0.030	0.954	0.939
Re-centered Gamma PML								
Analytical BC	-0.001	-0.023	0.915	0.926	0.0005	0.031	0.921	0.929
Bootstrap BC	-0.001	-0.024	0.926	0.935	0.0004	0.030	0.925	0.928
2-step Bootstrap BC	-0.001	-0.046	0.928	0.933	0.0003	0.021	0.877	0.913
FRW boot BC	-0.001	-0.019	0.930	0.933	0.0005	0.032	0.929	0.931
Split-panel Jackknife BC	-0.001	-0.022	0.867	0.909	0.0005	0.031	0.924	0.929
Node Jackknife BC	-0.000	-0.015	0.901	0.925	-0.0003	-0.019	0.919	0.927
Fully corrected Gamma PML (top 3 + selected others)								
SPJ + Jackknife SEs	-0.001	-0.022	1.002	0.950	0.0005	0.031	0.965	0.939
Node J. + Jackknife SEs	-0.000	-0.015	0.986	0.951	-0.0003	-0.019	0.960	0.939
Analytical + jackknife SEs	-0.001	-0.023	1.001	0.949	0.0005	0.031	0.963	0.938
Bootstrap + bootstrap SEs	-0.001	-0.024	0.928	0.932	0.0004	0.030	0.908	0.920
FRWB + FRWB SEs	-0.001	-0.019	0.855	0.906	0.0005	0.032	0.867	0.909
Analytical + HC2 SEs	-0.001	-0.023	0.934	0.935	0.0004	0.030	0.963	0.943
Uncorrected + boot. SEs	-0.001	-0.023	0.959	0.940	0.0004	0.030	0.937	0.933
Uncorrected + jack SEs	-0.001	-0.023	1.048	0.960	0.0004	0.030	0.996	0.948

Notes: 1,000 repetitions + 1,000 bootstrap trials per repetition. Model: $y_{ijt} = \exp(\alpha_i + \gamma_j + 0.5x_{ij})\omega_{ij}$. **Case 2 is the case where Gamma PML is correctly specified.**

Takeaways from Simulations

- Have also done preliminary simulations with the three-way gravity model estimated w/ PPML
- similar results, though not ready to share

Takeaways from Simulations

Best overall methods

- For correcting SEs *only*: re-sample bootstrap, HC2, jackknife*
- For correcting *both* point estimates and SEs:
  - jackknife or analytical re-centering + bootstrap SEs
  - bootstrap re-centering + bootstrap SEs
  - other combinations specific to each model + estimator

Other results

- DO NOT USE FRACTIONAL WEIGHT BOOTSTRAP!
- Jackknife SEs tend to be over-conservative, can be *wildly* over-conservative due to large variance
- Computationally efficient (2-step and 3-step) bootstrap variants work well.

Empirical application (3 way PPML)

For the empirical application, I use a three-way gravity model:

$$y_{ijt} = \exp\left(\alpha_{it} + \gamma_{jt} + \eta_{ij} + \beta FTA_{ijt}\right)\omega_{ijt}.$$

- Estimate with PPML (will have $1/N$ bias due to α_{it} and γ_{jt})
- Data: same as Weidner and Zylkin (Total trade for 165 countries, 1995-2015, every 5 years)

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- Data: same as Weidner and Zylkin (Total trade for 165 countries, 1995-2015, every 5 years)

	Estimate		**Standard Error**
PPML ($\widehat{\beta}$)	.0821	Cluster-Robust (CR1)	.0275
WZ analytical BC ($\widehat{\beta}_A$)	.0857	Weidner-Zylkin CR2	.0305
Avg. bootstrap estimate ($\widehat{\beta}_B$)	.0786	Weidner-Zylkin approx.	.0304
Bootstrap BC ($2\widehat{\beta} - \widehat{\beta}_B$)	.0856	Bootstrap SE	.0304
Bootstrap the analytical BC	.0818		

Last slide

Overall takeaways

- Bootstrap methods are effective for improving inference for PML gravity estimators
- *How* you bootstrap matters
  - “Fractional weight” bootstrap performs poorly
- k-step bootstrap offers computational efficiency

When would you want to use bootstrap for bias correction?

- Can correct SEs and point estimates using one procedure rather than two.
- Doesn’t require deriving/coding the analytical formula for the bias