

# Help file for `ge_gravity`

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## Title

`ge_gravity` - Solves a simple general equilibrium one sector Armington-CES trade model.

## Syntax

```
ge_gravity exp_id imp_id flows beta, theta(real) gen_w(varname) gen_X(varname) [options]
```

*exp\_id*, *imp\_id* are variables that respectively identify the origin and destination country associated with each observation. *flows* are the observed trade flows in the data for the year being used as the baseline for the counterfactual. *beta* is an input reflecting the “partial” change in trade, typically obtained as a coefficient from a prior gravity estimation. For more details, please see the “Background” section below.

## Description

`ge_gravity` solves for general equilibrium effects of changes in trade policies using a one sector Armington-CES trade model. It uses a simple fixed point algorithm that allows for fast computation. This approach, together with the implementation in Stata, makes this program ideal for bootstrapping confidence intervals for general equilibrium simulations based on prior gravity estimates of FTAs or other similar variables. Examples of references that conduct general equilibrium analysis based on FTA estimates in this way include Egger, Larch, Staub, & Winkelmann (2011), Anderson & Yotov (2016), and Baier, Yotov, & Zylkin (2019). Yotov, Piermartini, Monteiro, & Larch (2016) provide a detailed survey and introduction to the topic.

## Main Options

These options allow you to store results for general equilibrium changes in trade flows, welfare, and real wages as a result of the change in trade frictions. You may also vary how trade imbalances are treated in the model.

- |                              |  |
|------------------------------|--|
| <b>theta(real)</b>           | An input is required for the trade elasticity.   |
| <b><u>m</u>ultiplicative</b> | If trade is unbalanced in the data (as it usually is), the default is to treat the trade balance as an additive component of national expenditure (see below.) The “multiplicative” option instead supposes that national expenditure is a fixed multiple of national output, as in Anderson & Yotov (2016). |
| <b>gen_w(newvar)</b>         | Store the resulting change in welfare for each country (i.e., the new level of welfare divided by the old level of welfare). The value that is saved corresponds with the exporter’s change in welfare.  |
| <b>gen_X(newvar)</b>         | Store the new level of trade for each pair of countries.   |

**gen\_rw(newvar)** Store the change in real wage for each country (i.e., the new real wage divided by the old real wage). Note that this is generally different from the change in welfare unless if either trade is balanced or the “multiplicative” option is chosen. The value that is saved corresponds with the exporter’s change in real wage.

As of v. 1.1, it’s also possible to compute the constituent parts of the change in the real wage—the change in the nominal wage and the change in the price index—using **gen\_nw(newvar)** and **gen\_P(newvar)**. The saved values always correspond to the exporting country.

## Background

As a typical application, consider the following model for international trade flows:

$$X_{ij} = \frac{A_i w_i^{-\theta} \tau_{ij}^{-\theta}}{\sum_k A_k w_k^{-\theta} \tau_{kj}^{-\theta}} E_j. \quad (1)$$

$X_{ij}$  are international trade flows.  $i$  and  $j$  are indices for origin and destination.  $E_j \equiv \sum_i X_{ij}$  is total expenditure (also equal to the sum of the value of shipments across all origins).  $A_i$  is a measure of the level of technology in each origin  $i$ ,  $w_i$  is the production cost, and  $\tau_{ij} > 1$  is an iceberg trade cost. The model assumes that goods received from different origins are imperfectly substitutable and that the degree of substitutability is governed by  $\theta > 0$ , which serves as a trade elasticity. Labor,  $L_i$ , is assumed to be the only factor of production. Trade imbalances, treated as exogenously given, are accounted for as an additive component of expenditure. Thus, we can also write national expenditure as the sum of labor income and the national trade deficit/surplus:  $E_j = w_j L_j + D_j$ .

To obtain the *partial* effects of changes in trade policies on trade (i.e., holding all wages fixed), a standard approach is to estimate (1) using structural gravity. For example, if we have a panel of trading countries and we want to know the “average partial effect” of FTAs on trade, we can estimate the following “three-way” gravity regression:

$$X_{ijt} = \exp \left[ \ln \alpha_{it} + \ln \alpha_{jt} + \ln \alpha_{ij} + \beta \times FTA_{ijt} \right] + e_{ijt}, \quad (2)$$

As discussed in Yotov, Piermartini, Monteiro, & Larch (2016), the estimated  $\beta$  from this specification tells us how much trade would increase on average between any pair of countries that sign an FTA if we hold fixed all the endogenous variables in the model. Or, put in more structural terms, it tells us the direct trade impact of the reduction in trade barriers associated with an FTA:  $\beta = -\theta \Delta \ln \tau_{ijt}$ .

However, often the real goal is to compute *general equilibrium* changes in real wages, welfare, and/or trade volumes as a result of a change in trade frictions. In general equilibrium, the value of a country’s shipments across all destinations must add up to its labor income ( $Y_i \equiv w_i L_i = \sum_j X_{ij}$ .) In other words, we must have that

$$w_i L_i = \sum_j \frac{A_i w_i^{-\theta} \tau_{ij}^{-\theta}}{\sum_k A_k w_k^{-\theta} \tau_{kj}^{-\theta}} (w_j L_j + D_j) \quad \forall i. \quad (3)$$

This equation pins down each country’s wages (subject to a normalization) as a function of how easily it can

sell to markets with high levels of demand. Similarly, notice that we can express the effective price level in each country,  $P_j \equiv \left[ \sum_k A_k w_k^{-\theta} \tau_{kj}^{-\theta} \right]^{-1/\theta}$ , as a function of how easily a country can buy from producers with high technology levels and low production costs. These linkages are both intuitive and general; they can be found in many different trade models typically used for GE analysis.

A useful point about the system of equations in (3) is that it can be solved in “changes” (as opposed to solving it in levels). Adopting the notation of Dekle, Eaton, & Kortum (2007), let a “hat” over a variable denote the change in that variable resulting from a policy change (e.g.,  $\widehat{w}_i \equiv w'_i/w_i$  gives the change in  $i$ 's wage level.) Noting that  $\widehat{\tau}_{ij}^{-\theta} = e^{\beta \times FTA_{ij}}$ , the “equilibrium in changes” version of (3) can be written as:

$$Y_i \widehat{w}_i = \widehat{w}_i^{-\theta} \sum_j \frac{\pi_{ij} \cdot e^{\beta \times FTA_{ij}}}{\widehat{P}_j^{-\theta}} \cdot (Y_j \widehat{w}_j + D_j) \quad \forall i, \quad (4)$$

where  $\pi_{ij} \equiv X_{ij}/E_j$  is a bilateral trade share and  $\widehat{P}_j \equiv \left[ \sum_k \pi_{kj} \widehat{w}_k^{-\theta} e^{\beta \times FTA_{kj}} \right]^{-1/\theta}$  describes the change in price levels in each country. Notice this equation can be solved without knowledge of technology levels, endowments, initial trade frictions, or initial wages. All that is needed are trade volumes, national output and expenditure levels, and a value for the trade elasticity  $\theta$ .

Once changes in wages are known, GE changes in welfare, real wages, and trade volumes are given respectively by:

$$\text{GE Welfare Impact : } \widehat{W}_i = \widehat{E}_i / \widehat{P}_i \quad (5)$$

$$\text{GE Real Wage Impact : } r \widehat{w}_{ij} = \widehat{w}_i / \widehat{P}_i, \quad (6)$$

$$\text{GE Trade Impact : } \widehat{X}_{ij} = \frac{\widehat{w}_i^{-\theta} e^{\beta \times FTA_{ij}}}{\widehat{P}_j^{-\theta}} \cdot \widehat{E}_j, \quad (7)$$

where the change in national expenditure,  $\widehat{E}_i$ , is given by  $(Y_i \widehat{w}_i + D_i)/E_i$ . Because trade volumes are nominal quantities, there is one normalization needed. For this, the algorithm assumes that total world output is the same across both the baseline and the counterfactual (i.e.,  $\sum_i Y_i \widehat{w}_i = \sum_i Y_i$ .) The precise method used to solve the model is described further below.

## Examples

These examples follow the sample .dta and .do files that have been included along with this command. The data set consists of a panel of 44 countries trading with one another over the years 2000-2014. The trade data uses aggregated trade flows based on [WIOD](#) and information on FTAs is from the [NSF-Kellogg database](#) maintained by Scott Baier and Jeff Bergstrand.

Suppose the researcher wishes to use this data set to quantify general equilibrium trade and welfare effects of the EU enlargements that took place between 2000-2014. To first obtain the *partial* effects of these enlargements on trade flows, a PPML gravity specification may be used:

```
ppmlhdfc trade eu_enlargement other_fta if exporter != importer, a(expcode#year impcode#year
expcode#impcode) cluster(expcode#impcode)
```

This syntax implements the equivalent of (2) using the `ppmlhdfc` command created by Correia, Guimarães, & Zylkin (2019). In addition to estimating the effects of EU enlargements on new EU pairs, this example also controls for any other FTAs signed during the period. Each of these variables is coded as a dummy variable that becomes 1 when the agreement goes into effect for a given pair. The estimated coefficient for `eu_enlargements` is 0.224, implying that the expansion of the EU had an average partial effect of  $e^{0.224} - 1 = 25.1\%$  on trade between new EU members and existing members. With clustered standard errors, this estimate is statistically significant at the  $p < .01$  significance level.

Next, to obtain the general equilibrium effects of these new additions to the EU (as of 2000) we can input the following code:

```
sort exporter importer year
by exporter importer: gen new_eu_pair = (eu_enlargement[_N]-eu_enlargement[1])
gen eu_effect = _b[eu_enlargement] * new_eu_pair
ge_gravity exporter importer trade eu_effect if year==2000, theta(4) gen_w(w_eu) gen_X(X_eu)
```

This assumes a standard trade elasticity value of  $\theta = 4$ . The input for “*beta*” is given by a variable called `eu_effect`, which is equal to 0.224 for new EU pairs formed during the period and equal to 0 otherwise. Because of the small size of the sample, it solves almost instantly. Unsurprisingly, the new EU members (Bulgaria, Croatia, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Romania, Slovakia, and Slovenia) realize the largest welfare gains from their joining the EU, with existing EU countries also gaining. All countries not included in the agreement experience small losses due to trade diversion, with the largest losses accruing to Russia.

We can also change how trade imbalances enter the model. The default is to assume that they enter expenditure additively (i.e.,  $E_j = Y_j + D_j$ ), but one can also change the model so that expenditure is instead a fixed multiple of income (i.e., let  $E_j = \delta_j Y_j$ .) This is done using the multiplicative option:

```
ge_gravity exporter importer trade eu_effect if year==2000, theta(4) gen_w(w_eu) gen_X(X_eu)
mult
```

While using multiplicative imbalances instead of additive balances changes the results slightly, they are still qualitatively very similar.

An important point about the above exercises is that the initial partial effect is estimated with some uncertainty. The GE results that were calculated may paint a misleading picture because they do not take this uncertainty into account. For this reason, it is considered good practice to use a bootstrap method to construct confidence intervals for the GE calculations. This type of procedure is easily coded using `ge_gravity`. The included file “`GE_gravity_example.do`” provides a simple demonstration.

## Algorithm

While there are several ways to solve for counterfactuals in the above model, the simplest approach is arguably a fixed point algorithm that repeatedly iterates on the equilibrium conditions of the model. There are numerous ways to implement such an algorithm, but the approach used in Baier, Yotov, & Zylkin (2019) is especially simple to program. We first initialize  $\widehat{w}_i = \widehat{P}_i^{-\theta} = 1 \forall i$  and  $E'_i = E_i \forall i$ . The iteration loop then requires only 4 steps:

1. Update  $\widehat{w}_i \forall i$  one time using

$$\widehat{w}_i = \left[ Y_i^{-1} \sum_j \frac{\pi_{ij} \cdot e^{\beta \times FTA_{ij}}}{\widehat{P}_j^{-\theta}} \cdot E'_j \right]^{\frac{1}{1+\theta}} \quad \forall i.$$

2. Normalize all wages so that world output stays fixed:  $\sum_i Y_i \widehat{w}_i = \sum_i Y_i$ .
3. Update  $\widehat{P}_j^{-\theta} = [\sum_k \pi_{kj} \widehat{w}_k^{-\theta} e^{\beta \times FTA_{kj}}] \forall j$ .
4. Update  $E'_j = Y_j \widehat{w}_j + D_j \forall j$ .

(Repeat steps 1-4 until convergence.)

This algorithm is very similar to one previously made available by Head & Mayer (2014), but takes a slightly more streamlined approach to updating wages in step 1. It should generally be very fast because it does not involve using a nonlinear solver and because it has been implemented in Mata.

## Advisory

This is an advanced technique that requires a basic understanding of the model being solved. I would recommend reading either Section 4.3 of Head & Mayer (2014) or Ch. 2 of Yotov, Piermartini, Monteiro, & Larch (2016) before implementing.

One common issue that researchers new to these methods should be aware of is that GE trade models require a “square” data set with information on internal trade flows in addition to data on international trade flows. In the model, these internal flows are denoted by  $X_{ii}$ . If `ge_gravity` detects that the variable given for *flows* does not include one or more  $X_{ii}$  terms, it will exit with an error. Not all publicly available trade data sets include internal trade values. But some that do include [WIOD](#), [Eora MRIO](#), and the data set made available by [UNCTAD](#) as part of their online course on trade policy analysis (see Yotov, Piermartini, Monteiro, & Larch, 2016.)

This is version 1.1 of this command. Depending on interest, future versions could feature additional options such as allowances for tariff revenues and/or multiple sectors. If you believe you have found an error that can be replicated, or have other suggestions for improvements, please feel free to [contact me](#).

## Acknowledgements

The basic idea of using fixed point iteration to solve the gravity model has previously been implemented in Stata by Head & Mayer (2014) and Anderson, Larch, & Yotov (2015).

## Suggested citation

If you are using this command in your research, I would appreciate if you would cite

- Baier, Scott L., Yoto V. Yotov, and Thomas Zylkin. “On the widely differing effects of free trade agreements: Lessons from twenty years of trade integration.” *Journal of International Economics* 116 (2019): 206-226.

The algorithm used in this command was specifically written for the exercises performed in this paper. Section 6 of the paper provides a more detailed description of the underlying model and its connection to the literature.

## Further Reading

- Structural gravity: Anderson & van Wincoop (2003); Head & Mayer (2014)
- Methods for solving trade models: Alvarez & Lucas (2007); Anderson, Larch, & Yotov (2015); Head & Mayer (2014)
- Hat algebra: Dekle, Eaton, & Kortum (2007)
- GE effects of EU enlargements: Felbermayr, Gröschl, & Heiland (2018); Mayer, Vicard, & Zignago (2018).

## References

- Alvarez, F. & Lucas, J., Robert E. (2007), “General equilibrium analysis of the Eaton–Kortum model of international trade”, *Journal of Monetary Economics* **54**(6), 1726–1768.
- Anderson, J. E., Larch, M., & Yotov, Y. V. (2015), “Estimating General Equilibrium Trade Policy Effects: GE PPML”, *CESifo Working Paper* **5592**.
- Anderson, J. E. & van Wincoop, E. (2003), “Gravity with Gravitas: A Solution to the Border Puzzle”, *American Economic Review* **93**(1), 170–192.
- Anderson, J. E. & Yotov, Y. V. (2016), “Terms of trade and global efficiency effects of free trade agreements, 1990–2002”, *Journal of International Economics* **99**, 279–298.
- Baier, S. L., Yotov, Y. V., & Zylkin, T. (2019), “On the widely differing effects of free trade agreements: Lessons from twenty years of trade integration”, *Journal of International Economics* **116**, 206–226.
- Correia, S., Guimarães, P., & Zylkin, T. (2019), “ppmlhdf: Fast Poisson Estimation with High-Dimensional Data”, *arXiv preprint arXiv:1903.01690* .
- Dekle, R., Eaton, J., & Kortum, S. (2007), “Unbalanced Trade”, *American Economic Review* **97**(2), 351–355.
- Egger, P., Larch, M., Staub, K. E., & Winkelmann, R. (2011), “The Trade Effects of Endogenous Preferential Trade Agreements”, *American Economic Journal: Economic Policy* **3**(3), 113–143.
- Felbermayr, G., Gröschl, J. K., & Heiland, I. (2018), “Undoing Europe in a new quantitative trade model”, Tech. rep., Ifo Working Paper.

- Head, K. & Mayer, T. (2014), “Gravity Equations: Workhorse, Toolkit, and Cookbook”, in G. Gopinath, E. Helpman, & K. Rogoff (eds.) *Handbook of International Economics*, vol. 4, pp. 131–195, North Holland, 4 ed.
- Mayer, T., Vicard, V., & Zignago, S. (2018), “The cost of non-Europe, revisited”, .
- Yotov, Y. V., Piermartini, R., Monteiro, J.-A., & Larch, M. (2016), *An Advanced Guide to Trade Policy Analysis: The Structural Gravity Model*, World Trade Organization, Geneva.